

Examiners' Report: Final Honour School of Mathematics Part C Trinity Term 2016

October 20, 2016

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

See Table 1, page 1.

- **Numbers of vivas and effects of vivas on classes of result.**

As in previous years there were no vivas conducted for the FHS of Mathematics Part C.

- **Marking of scripts.**

The dissertations and mini-projects were double marked. The remaining scripts were all single marked according to a pre-agreed marking scheme which was very closely adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**

See Table 7 on page 8.

Table 1: Numbers in each class

	Number					Percentages %				
	2016	(2015)	(2014)	(2013)	(2012)	2016	(2015)	(2014)	(2013)	(2012)
I	44	(45)	(45)	(56)	(45)	50.57	(46.39)	(45.92)	(47.46)	(45.45)
II.1	31	(39)	(42)	(41)	(36)	35.63	(40.21)	(42.86)	(34.75)	(36.36)
II.2	9	(13)	(11)	(15)	(15)	10.34	(13.4)	(11.22)	(12.71)	(15.15)
III	3	(0)	(0)	(4)	(3)	3.45	(0)	(0)	(3.39)	(3.03)
F	0	(0)	(0)	(2)	(0)	0	(0)	(0)	(1.69)	(0)
Total	87	(97)	(98)	(118)	(99)	100	(100)	(100)	(100)	(100)

B. New examining methods and procedures

For the first time, two mathematics courses were assessed by mini-project: Networks and Computational Algebraic Topology.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

Two changes to examining procedures have been agreed for next year. Firstly, the length of time allowed for mathematics unit papers will increase from 1.5 hours to 1.75 hours. Secondly, the supervisors of dissertations will now be formally appointed as one of the two assessors for the project.

D. Notice of examination conventions for candidates

The first notice to candidates was issued on 16th February 2016 and the second notice on 28th April 2016. These contain details of the examinations and assessments.

All notices and the examination conventions for 2016 examinations are on-line at <http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments>.

Part II

A. General Comments on the Examination

The examiners would like to thank in particular Helen Lowe, Waldemar Schlackow and Nia Roderick for their commitment and dedication in running the examination systems. We would also like to thank Charlotte Turner-Smith, and the rest of the Academic Administration Team for all their work during the busy exam period.

We also thank the assessors for their work in setting questions on their own courses, and for their assistance in carefully checking the draft questions of other assessors, and also to the many people who acted as assessors for dissertations. We are particularly grateful to those—this year the great majority—who abided by the specified deadlines and responded promptly to queries. This level of cooperation contributed in a significant way to the smooth running of what is of necessity a complicated process.

The internal examiners would like to thank the external examiners Professor Chris Howls and Professor Alexei Skorobogatov for their prompt and careful reading of the draft papers and for their valuable input during the examiners' meeting.

Timetable

The examinations began on Monday 30th May and finished on Monday 13th June.

Setting and checking of papers and marks processing

Following established practice, the questions for each paper were initially set by the course lecturer, with the lecturer of a related course involved as checker before the first draft of the questions was presented to the examiners. The course lecturers also acted as assessors, marking the questions on their course(s).

The internal examiners met in early January to consider the questions on Michaelmas Term courses, and changes and corrections were agreed with the lecturers where necessary. The revised questions were then sent to the external examiners. Feedback from external examiners was given to examiners, and to the relevant assessor for each paper for a response. The internal examiners met a second time late in Hilary Term to consider the external examiners' comments and assessor responses (and also Michaelmas Term course papers submitted late). The cycle was repeated for the Hilary Term courses, with two examiners' meetings in the Easter Vacation; the schedule here was much tighter. Following the preparation of the Camera Ready Copy of the papers as finally approved, each assessor signed off their paper in time for submission to Examination Schools in week 1 of Trinity Term.

A team of graduate checkers, under the supervision of Helen Lowe, sorted all the marked scripts for each paper of this examination, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way a number of errors were corrected, each change was signed by one of the examiners who were present throughout the process. A check-sum is also carried out to ensure that marks entered into the database are correctly read and transposed from the marks sheets.

Determination of University Standardised Marks

The Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average in each class over the last four years, together with recent historic data for Part C, the MPLS Divisional averages, and the distribution of classifications achieved by the same group of students at Part B.

The examiners followed established practice in determining the University standardised marks (USMs) reported to candidates. This leads to classifications awarded at Part C broadly reflecting the overall distribution of classifications which had been achieved the previous year by the same students.

We outline the principles of the calibration method.

The Department's algorithm to assign USMs in Part C was used in the same way as last year for each unit assessed by means of a traditional written examination. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part B classification of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage). Working with the data for this population, numbers N_1 , N_2 and N_3 are first computed for each paper: N_1 , N_2 and N_3 are, respectively, the number of candidates taking the paper who achieved in Part B overall

average USMs in the ranges $[70, 100]$, $[60, 69]$ and $[0, 59]$, respectively.

The algorithm converts raw marks to USMs for each paper separately (in each case, the raw marks are initially out of 50, but are scaled to marks out of 100). For each paper, the algorithm sets up a map $R \rightarrow U$ ($R = \text{raw}$, $U = \text{USM}$) which is piecewise linear. The graph of the map consists of four line segments: by default these join the points $(100, 100)$, $P_1 = (C_1, 72)$, $P_2 = (C_2, 57)$, $P_3 = (C_3, 37)$, and $(0, 0)$. The values of C_1 and C_2 are set by the requirement that the proportion of I and II.1 candidates in Part B, as given by N_1 and N_2 , is the same as the I and II.1 proportion of USMs achieved on the paper. The value of C_3 is set by the requirement that P_2P_3 continued would intersect the U axis at $U_0 = 10$. Here the default choice of *corners* is given by U -values of 72, 57 and 37 to avoid distorting nonlinearity at the class borderlines.

The results of the algorithm with the default settings of the parameters provide the starting point for the determination of USMs. The examiners have scope to make changes, usually by adjusting the position of the corner points P_1, P_2, P_3 by hand, so as to alter the map $\text{raw} \rightarrow \text{USM}$, to remedy any perceived unfairness introduced by the algorithm, in particular in cases where the number of candidates is small. They also have the option to introduce additional corners.

Table 2 on page 5 gives the final positions of the corners of the piecewise linear maps used to determine USMs from raw marks. For each paper, P_1, P_2, P_3 are the (possibly adjusted) positions of the corners above, which together with the end points $(100, 100)$ and $(0, 0)$ determine the piecewise linear map $\text{raw} \rightarrow \text{USM}$. The entries N_1, N_2, N_3 give the number of incoming firsts, II.1s, and II.2s and below respectively from Part B for that paper, which are used by the algorithm to determine the positions of P_1, P_2, P_3 .

Following customary practice, a preliminary, non-plenary, meeting of examiners was held two days ahead of the plenary examiners' meeting to assess the results produced by the algorithm alongside the reports from assessors. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low. These revised USM maps provided the starting point for a review of the scalings, paper by paper, by the full board of examiners.

Table 2: Position of corners of piecewise linear function

Paper	P_1	P_2	P_3	Additional corners	N_1	N_2	N_3
C1.1	(14.59,37)	(25.4,57)	(43.4,72)		4	4	3
C1.2	(12.24,37)	(21.3,57)	(33,72)		3	4	2
C1.3	(8.96,37)	(15.6,57)	(27.6,72)		9	8	2
C1.4	(9.77,37)	(17,57)	(32,72)		5	9	3
C2.1	(13.04,37)	(22.74,57)	(33.5,72)		8	0	1
C2.2	(9.77,37)	(17,57)	(32,72)		9	5	1
C2.3	(18.79,37)	(33,70)			3	0	0
C2.4	(8,37)	(17,57)	(25,72)		2	3	0
C2.5	(16.49,37)	(28,57)	(39,72)		5	0	0
C2.6	(21.37,37)	(33,57)	(45,70)		4	2	0
C2.7	(7.7,37)	(19,57)	(35.5,72)		8	6	1
C3.1	(11.55,37)	(20.1,57)	(28,72)		5	4	0
C3.2	(9.59,37)	(17,57)	(29,72)		5	0	0
C3.3	(14.42,37)	(20,57)	(33,72)		5	2	0
C3.4	(11.89,37)	(27,57)	(37.2,72)		7	7	0
C3.5	(10,37)	(23,57)	(35.4,72)		3	1	1
C3.6	(14.99,37)	(26.5,57)	(41,72)		4	7	0
C3.7	(12,37)	(24,57)	(37,72)		8	9	1
C3.8	(10.34,37)	(24,57)	(34,72)		7	11	1
C4.1	(6.32,37)	(22,57)	(36,72)		7	9	1
C4.2	(8.5,37)	(19,57)	(30,72)		5	7	0
C4.3	(6.15,37)	(20,57)	(30,72)		4	3	0
C4.4	(11.32,37)	(22,57)	(30,72)		3	0	0
C4.5	(10.57,37)	(23,57)	(36.4,72)		4	6	0
C4.6					3	0	0
C4.7					1	2	0
C5.1	(15,37)	(29,57)	(40,70)		2	14	0
C5.2	(8,37)	(25,57)	(36,72)		5	12	0
C5.3	(10.51,37)	(18.3,57)	(32,72)		1	4	0
C5.4	(23,37)	(56,57)	(70,72)		7	9	1
C5.5	(8,37)	(21,57)	(35,72)		7	16	1
C5.6	(11.6,37)	(22,57)	(31,72)		7	6	0
C5.7	(10.4,37)	(19,57)	(37.6,72)		6	10	0
C5.9	(10,37)	(22,57)	(36,72)		1	7	0
C5.11	(12,37)	(19,57)	(35,72)		2	7	0
C5.12	(12.29,37)	(23,57)	(36,72)		4	16	2
C6.1	(8.67,37)	(25,57)	(39,72)		2	9	3
C6.2	(10.46,37)	(21,57)	(36,72)		2	10	3
C6.3	(17,37)	(27,57)	(40,70)		1	11	4

Paper	P_1	P_2	P_3	Additional corners	N_1	N_2	N_3
C6.4	(17.41,37)	(31,57)	(40,70)		6	13	4
C7.3	(13,37)	(23.8,57)	(36,72)		1	4	0
C7.4	(18.44,37)	(32.1,57)	(40,70)		0	6	1
C7.5	(16.09,37)	(30,57)	(39,72)		1	3	0
C7.6	(13.44,37)	(25,57)	(37,72)		1	3	0
C8.1	(16.2,37)	(28.2,57)	(37.2,72)		7	3	0
C8.2	(13,37)	(22,57)	(35,72)		6	2	0
C8.3	(12.52,37)	(25,57)	(40,70)		13	17	4
C8.4	(10.23,37)	(22,57)	(38,72)		14	13	5
SC1	(14.25,37)	(26,57)	(40,70)		9	13	1
SC2	(10.91,37)	(23,57)	(42,72)		4	15	2
SC3	(11.2,37)	(23,57)	(32,72)		1	5	0
SC4	(10.91,37)	(21,57)	(30,72)		3	13	3
SC5	(7.58,37)	(20,57)	(35,72)		3	13	1

Table 4 on page 6 gives the rank of candidates and the number and percentage of candidates attaining this or a greater (weighted) average USM.

Table 4: Percentile table for overall USMs

Av USM	Rank	Candidates with this USM or above	%
92	1	1	1.15
91	2	2	2.3
88	3	3	3.45
84	4	6	6.9
83	7	10	11.49
82	11	12	13.79
79	13	13	14.94
77	14	16	18.39
76	17	20	22.99
75	21	22	25.29
74	23	28	32.18
73	29	35	40.23
72	36	39	44.83
71	40	40	45.98
70	41	44	50.57
69	45	46	52.87
68	47	48	55.17
67	49	52	59.77
66	53	56	64.37
65	57	61	70.11
64	62	66	75.86
63	67	68	78.16
62	69	73	83.91
61	74	74	85.06

Av USM	Rank	Candidates with this USM or above	%
60	75	75	86.21
59	76	76	87.36
58	77	79	90.8
57	80	80	91.95
56	81	82	94.25
54	83	83	95.4
53	84	84	96.55
49	85	86	98.85
44	87	87	100

B. Breakdown of the results by gender

Table 6, on page 7 shows the performances of candidates broken down by gender.

Table 6: Breakdown of results by gender

Class	Total		Female		Male	
	Number	%	Number	%	Number	%
I	44	50.57	10	41.67	34	53.97
II.1	31	35.63	10	41.67	21	33.33
II.2	9	10.34	4	16.67	5	7.94
III	3	3.45	0	0	3	4.76
F	0	0	0	0	0	0
Total	87	100	24	100	63	100

C. Detailed numbers on candidates' performance in each part of the exam

Data for papers with fewer than six candidates are not included.

Table 7: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
C1.1	11	33.91	11.4	65.45	17.53
C1.2	9	27.44	10.25	63	18.49
C1.3	19	25.68	6.97	69.21	9.91
C1.4	17	26.47	7.05	66.88	8.51
C2.1	9	34.22	9.83	73.67	16.29
C2.2	15	31.2	9.52	72.07	14.15
C2.3	3	-	-	-	-
C2.4	6	14.67	8.98	47.83	23.79
C2.5	5	-	-	-	-
C2.6	6	45.17	3.71	77.67	12.09
C2.7	15	30.2	10.75	67.4	14.76
C3.1	10	24.8	4.83	65.3	7.39
C3.2	6	23.17	5.27	64.83	6.62
C3.3	7	35.57	9.14	77.57	13.75
C3.4	14	36.29	8.04	72.71	14.66
C3.5	5	-	-	-	-
C3.6	12	38.83	6.93	72.42	11.65
C3.7	19	33.84	6.99	69.11	10.2
C3.8	20	31.25	8.1	68.55	13
C3.9	2	-	-	-	-
C4.1	16	31.75	9.31	68.81	12.01
C4.2	12	23.92	10.67	62.67	16.23
C4.3	6	24.33	11.36	63.33	16.49
C4.4	3	-	-	-	-
C4.5	10	32.8	8.13	69.5	12.27
C4.6	3	-	-	-	-
C4.7	3	-	-	-	-
C5.1	16	36.56	10.15	69.19	17.29
C5.2	17	33.47	7.68	69.82	11.91
C5.3	5	-	-	-	-
C5.4	18	62.06	14.15	64.28	12.89
C5.5	22	30.91	7.44	68.86	10.37
C5.6	13	30.62	6.17	70.92	9.56
C5.7	16	32.5	7	69.25	8.13
C5.9	8	32	7.71	68.62	9.93
C5.11	9	29.22	8.71	65.56	12.24
C5.12	22	31.18	5.46	66.86	7.67
C6.1	13	31.92	12	66.23	16.16
C6.2	16	27.69	7.36	64	8.77
C6.3	16	36.38	8.02	70.19	14.16

Paper	Number of Candidates	Avg	StDev	Avg	StDev
		RAW	RAW	USM	USM
C6.4	24	38.38	5.59	69.83	11.05
C7.1	2	-	-	-	-
C7.3	4	-	-	-	-
C7.4	5	-	-	-	-
C7.5	4	-	-	-	-
C7.6	4	-	-	-	-
C8.1	8	36.75	4.8	72.25	8.92
C8.2	6	31.67	9.44	69.5	14.11
C8.3	31	35.1	8.58	68.71	13.09
C8.4	27	31.56	9.19	66.63	12.18
SC1	13	40.23	6.52	76	12.01
SC2	10	31.4	9.67	66.4	14.01
SC3	1	-	-	-	-
SC4	9	27.56	4.5	67.67	7.47
SC5	4	-	-	-	-
CCD	32	-	-	74.69	6.354
COD	7	-	-	74	9.06
CCS1	2	-	-	-	-
CCS2	1	-	-	-	-
CCS3	3	-	-	-	-

The tables that follow give the question statistics for each paper for Mathematics candidates. Data for papers with fewer than six candidates are not included.

Paper C1.1: Model Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.9	15.9	6.9	10	0
Q2	7	7		1	0
Q3	18.82	18.82	4.92	11	0

Paper C1.2: Gödel's Incompleteness Theorems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11	11	4.24	4	0
Q2	15.67	15.67	6.63	9	0
Q3	12.4	12.4	5.37	5	0

Paper C1.3: Analytic Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.05	13.05	3.99	19	0
Q2	11.78	12.41	4.21	17	1
Q3	14.5	14.5	12.02	2	0

Paper C1.4: Axiomatic Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.57	13.57	4.93	14	0
Q2	10.83	10.83	5.04	6	0
Q3	13.93	13.93	3.85	14	0

Paper C2.1: Lie Algebras

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.71	17.71	6.05	7	0
Q2	16.29	16.29	4.61	7	0
Q3	14	17.5	8.92	4	1

Paper C2.2: Homological Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13	15.6	7.85	5	1
Q2	13.08	14.45	5.28	11	2
Q3	16.5	16.5	5.81	14	0

Paper C2.4: Infinite Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	8.67	8.67	6.06	6	0
Q2	6.8	6.8	3.11	5	0
Q3	2	2		1	0

Paper C2.6: Introduction to Schemes

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	23	23	3.08	5	0
Q2	22	22		1	0
Q3	22.33	22.33	2.5	6	0

Paper C2.7: Category Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.9	14.78	6.72	9	1
Q2	16	16	4.76	10	0
Q3	13.31	14.55	6.74	11	2

Paper C3.1: Algebraic Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.11	11.57	6.17	7	2
Q2	9.17	10.25	3.25	4	2
Q3	14	14	2.35	9	0

Paper C3.2: Geometric Group Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16	16	3.61	3	0
Q2	11.8	11.8	3.35	5	0
Q3	8	8	2.71	4	0

Paper C3.3: Differentiable Manifolds

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16	16	6.48	4	0
Q2	19.4	22.75	8.02	4	1
Q3	15.67	15.67	5.35	6	0

Paper C3.4: Algebraic Geometry

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18	18	5.26	14	0
Q2	18.63	18.63	4.24	8	0
Q3	17.83	17.83	2.64	6	0

Paper C3.6: Modular Forms

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.6	18.6	4.97	10	0
Q2	19.7	19.7	4.47	10	0
Q3	18.4	20.75	5.81	4	1

Paper C3.7: Elliptic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.47	19.47	3.59	17	0
Q2	12.69	13.58	6.2	12	1
Q3	16.56	16.56	3.81	9	0

Paper C3.8: Analytic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.44	17.44	3.93	18	0
Q2	14.77	16	6.17	12	1
Q3	11.9	11.9	6.03	10	0

Paper C4.1: Functional Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.06	20.33	7.24	15	1
Q2	12.5	12.5	6.76	12	0
Q3	10.6	10.6	4.22	5	0

Paper C4.2: Linear Operators

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.33	13.33	5.69	12	0
Q2	12.22	12.22	4.87	9	0
Q3	8.5	8.5	4.95	2	0

Paper C4.3: Functional Analytical Methods for PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.67	15.67	4.41	6	0
Q2	9	9	8.29	4	0
Q3	8	16	11.31	1	1

Paper C4.5: Ergodic Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.6	19.6	5.27	10	0
Q2	14.71	14.71	3.95	7	0
Q3	9.5	9.67	4.51	3	1

Paper C5.1: Solid Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.53	19.23	7.81	13	2
Q2	14.17	19.13	7.93	8	4
Q3	15.85	16.55	5.93	11	2

Paper C5.2: Elasticity and Plasticity

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.5	15.33	7.72	3	1
Q2	16.53	17.56	5.22	16	1
Q3	16.13	16.13	4.45	15	0

Paper C5.5: Perturbation Methods

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.39	15.06	5.7	17	1
Q2	15.71	16.25	4.63	16	1
Q3	13.75	14.91	4.61	11	1

Paper C5.6: Applied Complex Variables

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.33	17.33	1.8	9	0
Q2	12.38	12.38	3.54	8	0
Q3	14.7	15.89	6.38	9	1

Paper C5.7: Topics in Fluid Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.14	15.14	2.04	7	0
Q2	15.21	15.21	4.15	14	0
Q3	18.27	18.27	4.36	11	0

Paper C5.9: Mathematical Mechanical Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16	20	9.62	4	1
Q2	12.2	12.2	5.45	5	0
Q3	16.43	16.43	2.99	7	0

Paper C5.11: Mathematical Geoscience

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.5	11.5	4.8	4	0
Q2	17.22	17.22	4.76	9	0
Q3	12.4	12.4	6.43	5	0

Paper C5.12: Mathematical Physiology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.78	16.78	2.41	18	0
Q2	14.85	14.85	3.48	13	0
Q3	14.69	14.69	4.09	13	0

Paper C6.1: Numerical Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.82	14.8	5.76	10	1
Q2	17.91	17.91	7.45	11	0
Q3	12.29	14	6.24	5	2

Paper C6.2: Continuous Optimization

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.38	14.38	4.06	16	0
Q2	12.67	12.67	6.74	6	0
Q3	13.09	13.7	4.64	10	1

Paper C6.3: Approximation of Functions

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.2	19.2	4.48	15	0
Q2	17.27	18.77	5.57	13	2
Q3	12.2	12.5	3.7	4	1

Paper C6.4: Finite Element Methods for Partial Differential Equations

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.81	19.74	4.26	19	2
Q2	18.62	18.6	3.06	20	1
Q3	17.4	19.33	7.43	9	1

Paper C8.1: Stochastic Differential Equations

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.5	15.5	3	4	0
Q2	17.25	17.25	2.22	4	0
Q3	20.38	20.38	4.14	8	0

Paper C8.2: Stochastic Analysis and PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.25	15.25	7.37	4	0
Q2	14.8	17	6.42	4	1
Q3	15.25	15.25	5.74	4	0

Paper C8.3: Combinatorics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.6	18.6	4.58	30	0
Q2	16.74	16.74	4.91	23	0
Q3	16.11	16.11	4.91	9	0

Paper C8.4: Probabilistic Combinatorics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.48	15.48	4.76	27	0
Q2	16.27	16.27	4.83	26	0
Q3	11	11		1	0

Paper SC1: Stochastic Models in Mathematical Genetics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20	20	2.96	9	0
Q2	21.36	21.36	2.69	11	0
Q3	18	18	5.1	6	0

Paper SC2: Probability and Statistics for Network Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.56	15.56	5.68	9	0
Q2	14.6	18	8.25	8	2
Q3	8.75	10	3.3	3	1

Paper SC4: Statistical Data Mining and Machine Learning

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18	18	2	5	0
Q2	11.71	12.67	3.55	6	1
Q3	11.5	11.71	2.51	7	1

Paper SC5: Advanced Simulation Methods

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.25	16.25	3.1	4	0
Q2	12	12	4.24	2	0
Q3	10.5	10.5	2.12	2	0

D. Recommendations for Next Year's Examiners and Teaching Committee

This year there was a slightly higher than average percentage of students getting a first. There were also more first class students coming in which seems to be one of the main reasons for this. This was noted by the examiners and no recommendations were made.

It was noted by the examiners that several of the exams had only a few candidates taking them. This can cause difficulties in producing the USM maps. On the other hand, these courses are attended by graduate students and there seems to be a clear pedagogical reason to give these courses. Also in light of the plans to have a stand alone fourth year, the number of part C courses currently given seems to be right. This was noted by the examiners and no recommendations were made.

E. Comments on papers and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. Some data to be found in Section C above have been omitted.

C1.1: Model Theory

In all three questions a substantial part was a 'bookwork' and 'seen' -type problems, so the candidates could score 18-19 out of 25 marks.

Question 1 was of a more standard kind but the unseen part was more tricky.

The unseen part of question 2 apparently looked the most difficult for the candidates. In fact, it was based on one of the basic examples considered in the course and they should have seen a lot of it in the exercises.

The unseen part of question 3 proved to be the easiest of the three and so candidates with a good understanding of the material were able to score high marks.

C1.2: Gödel's Incompleteness Theorems

Question 1. Generally speaking, it was inadequately done. 1(a)(i) was bookwork, as was essentially 1(a)(ii); some candidates offered too brief a sketch in answer to the latter. 1(a)(iii) was unseen, though discussed in passing in lectures; the argument that the standard model embeds into any model of PA was not produced by any candidate. 1(a)(iv) was bookwork and was generally well, though not perfectly, done. 1(b) was essentially bookwork from the trickier part of the course, and poorly done. For a counterexample to 1(b)(iii), a sentence (as opposed to an open formula) would have sufficed.

Question 2, the easiest of the three. Parts (a) and (b) were bookwork and were generally very well done. Part (d) was also bookwork, which candidates did not on the whole seem to have assimilated. Part (c)(i) was an exercise on the lecture notes and proved easy for some candidates, difficult for others. Part (c)(ii) was the only part of the question that candidates would not have seen before, but had a straightforward answer: let X be a Gödel sentence for S .

Question 3. Parts (a) and (b) were bookwork and were almost uniformly perfectly answered. Parts (c) and (d) proved to be surprisingly difficult, with few candidates availing themselves of the result stated in (a). Part (e) also proved difficult; although it offered an opportunity to deploy the algorithms described in the last part of the course, it was also susceptible to a very short proof, spotted by some candidates.

C1.3: Analytic Topology

Questions 1 and 2 were very popular, whilst only very few candidates attempted question 3.

Question 1 The bookwork was generally well done, although a lot of candidates missed out different parts of this. A common mistake was to assume that the diagonal image of dense embeddings was dense.

A number candidates had good ideas for the second part, although very few managed to give an entirely correct answer. A common mistake was to assume that if f is a continuous bounded real-valued map on X then $(f, \overline{f(X)})$ is a compactification, although of course f might not be injective. However, following the proofs from the first part, this approach would yield a proof that the two compactifications are equivalent, so a generous amount of partial marks were given.

Surprisingly, not many candidates even attempted the last part of the question and from the few that did, some chose to manually prove non-equivalence instead of appealing to the previous part.

Question 2 Again, the bookwork was done well, although the very last bit of part (a) had many mistakes where candidates did not ensure that each element of the refinement was in exactly one union.

In part (b), both directions were more challenging than expected. For the forwards direction, candidates successfully applied the method from the first part, but then did not know what

to do with the closed sets obtained. A few tried to use regularity, but here the problem is that the space is only countably paracompact. For the backwards part, a lot of candidates incorrectly claimed that the interiors of the C_n formed a locally finite family. Only very few candidates considered the complements of the sets obtained, but could then finish the proofs.

Most candidates did not attempt part (c), not even writing down the dual of the condition from part (b).

Question 3 Only very few candidates attempted this question. generally obtaining good marks. As expected and despite having seen the sup-norm since their first year, existence of the metric from part (b) was more difficult than showing its uniqueness which was done well.

C1.4: Axiomatic Set Theory

Questions 1 and 3 were a lot more popular than 2.

Question 1 Most of the bookwork was done well, although a number of candidates claimed that being a well-order could be expressed by a Δ_0 -formula. In part (b) when showing that the ordinals do not Pairing, a lot of candidates failed to mention that the unordered pair is absolute. Similarly for Separation, a lot of candidates showed that some set which ‘should’ exist, is not an ordinal, but failed to explain how this works when relativized to the ordinals. In part (c), there were a number of good answers, but also some problems with candidates trying to recurse on classes or intersecting a collection of classes.

Question 2 Of the few students attempting this, there were two very good answers, with the remaining ones mostly struggling with anything beyond part (a). With the exception of the two very good answers, candidates were generally unwilling to skip earlier parts and attempt later ones which might be easier.

Question 3 Both part (a) and (b) were mostly well done. Extensionality in (c) was straightforward, but already Pairing posed a lot of difficulties, with a number of obviously incorrect candidates for the unordered pair being given. Only very few candidates managed to fully show that (V, E) satisfies Union, but there were a number of reasonable attempts which were marked generously. Again, the main difficulty seemed to be identifying a suitable candidate. Only one candidate then had the insight to construct $\omega^{(V, E)}$ using the Pairing and Union axiom in (V, E) , together with recursion (in (V, \in)). Other attempts to write down $\omega^{(V, E)}$ explicitly were generally unsuccessful.

A good number of candidates attempting part (d) were able to show that Foundation fails in a suitable (V, E) , usually by constructing an F and some x such that $yEx \leftrightarrow y = x$. Surprisingly, no-one then noticed that this element satisfies the formula from (b).

C2.1: Lie Algebras

Most candidates attempted and solved satisfactorily Problem 1. A small number of the candidates had difficulties with part (c) and with part (e).

In problem 2, part (e) was the most difficult, only two candidates solved this part. The simplest solution is to show that the eigenvalues of each $\text{ad}(a)$ on the toral subalgebra are all zero; here a is an arbitrary element of the toral subalgebra.

In problem 3, there were some difficulties in finding a base for the root system in (d) and in the computation of the Cartan decomposition in (e). But overall, fewer candidates attempted this problem than the previous two.

C2.2: Homological Algebra

C2.3: Representation Theory of Semisimple Lie Algebras

The candidates attempted and solved satisfactorily Problems 1 and 3. The candidates had some difficulties with 1(e) and particularly with 3(e), where no correct solution was given. The easiest solution in 3(e) is to study the weight decomposition of the tensor product.

C2.4: Infinite Groups

All the questions were too long, and q.3 was too challenging.

Questions 1 and 2 were started off with some success by most of the candidates, but they quickly got stuck. No candidate completed two questions, one candidate completed q 2.

C2.5: Non-Commutative Rings

Questions 1 and 2 were universally popular and Question 3 universally unpopular with only one attempt. Some people forgot that a simple ring may have some proper non-trivial left ideals (e.g. $M_2(\mathbb{C})!$) and consequently lost marks in Question 1c. Only a couple of students managed to solve Question 2d fully.

C2.6 Introduction to Schemes

Q1. Most students had little difficulty with this question.

Q2. This question was attempted by very few students.

Q3. There are various ways to approach this question and most of them appeared in the solutions proposed by the students. It is important here not to forget that closed subsets of $\text{Spec } R$ are in 1 – 1 correspondence with **radical** ideals.

C2.7 Category Theory

Question 1. Parts (a) and (b) of the question were bookwork and most of the students did well once they realized that part (b) was about free algebras. Part (c) was clearly the

most difficult, but several people who attempted to do part (d) skipping part (c) were still partially successful.

Question 2. Part (a) of the question was standard bookwork. In part (b) a few people wrote about colimits of presheaves viewed as diagrams of sets rather than colimits of diagrams of presheaves. Many students remembered the shape of the colimit to present a general presheaf as a colimit of representable ones, but only one person gave a complete proof. Those who tried to do part (c) gave a correct construction of the left adjoint which did not use the previous parts of the question.

Question 3. Part (a) was again bookwork and mostly done well. Many students gave interesting and non-standard examples of limits and colimits which do not commute. Parts (b) and (c) were similar to the homework and many students recalled the general flow of the argument.

C3.1: Algebraic Topology

Question 1. There was one answer to this question receiving full marks. (a) In part (iii), there were some correct answers, though a number of candidates wrote down the first map they saw from A_n to C_{n+1} , namely $H\partial K$, where H and K were the two chain homotopies, without seeing that that map is not a chain homotopy. (b) Some candidates saw that the chain complex of the real projective plane provided a suitable example for both part (i) and part (ii). (c) Most candidates who attempted this part were able to compute the set of chain homotopy classes.

Question 2. (a) Most candidates did a reasonable job with this bookwork part. (b) Most candidates correctly set up the long exact sequence for the pair, but almost everyone failed to correctly identify the boundary map in the sequence as $1 - \det f$ —the geometry of the mapping torus evidently has two components to the boundary attachment, corresponding to the identity and to the induced map of f , taken with opposite orientations. (c) Some candidates correctly identified whether M_f and M_g were manifolds, but no one determined whether there was a manifold homotopy equivalent to M_g , despite this being an immediate consequence of parts (a.i) and (b.ii).

Question 3. Note that this was a minor variation on a question that appeared in the exam last year, and in the exam the year before, and in the problem sheet this year. Candidates are encouraged to ensure they confidently know how to thoroughly do all problems from all the problem sheets and past exams in recent years. (a) All candidates gave a reasonable answer to this bookwork part. (b) A number of candidates correctly presented the CW structure, but there were a number of sloppy mistakes, for instance identifying each of the two halves of the boundary circle of M_2 with the whole diagonal circle of M_1 . Most candidates correctly set up the Mayer–Vietoris sequence for computing the homology, and a few candidates managed to understand the geometry of the situation and therefore correctly identify the key map in the sequence as $(2, 1)$ (though some mixed up the two factors, leading to incorrect answers later). One or two candidates managed, more or less, to see through the key computation of $H_1(E)$. (c) Most candidates saw to apply the Universal Coefficient Theorem, but a combination of mistakes from part (b) and some incorrect Tor calculations, lead to few correct answers here.

C3.2 Geometric Group Theory

C3.3: Differentiable Manifolds

The last parts of all three questions caused difficulty for most candidates, but were all solved by some.

C3.4: Algebraic Geometry

All students chose Q1, and there was an almost even split between students choosing Q2 or Q3. Nobody attempted more than two questions. Average scores on the three questions were roughly equal, of a high standard: 18/25.

Q1) Failed attempts at (e) occurred from considering the ideal (p_1, \dots, p_m, r) instead of (p_1, \dots, p_m) . In (f) students often pointed out that X was only unique up to isomorphism, but they did not justify why X as constructed was not unique.

Q2) There was a typo: *rational functions* should say *regular functions*, but nobody was confused by this, presumably because the symbols clarified the meaning. Several students erroneously stated that regular functions on U are a ratio of functions globally on all of U , rather than just locally.

Q3) A few students got confused in (c), and tried to set up a projection map (instead of considering the P^n that records the coefficients of the linear form). Some students forgot the standard example to the last part of (f).

C3.5: Lie Groups

Question 3, when attempted, was poorly done.

C3.6: Modular Forms

Question 1

This was a challenging and quite long question, requiring a lot of original thinking in the later parts. It was done quite well, with several students getting full or nearly full marks.

Question 2

This question was a little more routine, involving a lot of bookwork in part (a) — the latter parts of (b) were original though and quite a nice application of the valence formula. Overall it was done well.

Question 3

Less than half of the candidates attempted this question, probably because it was on material on the later parts of the course. Most who attempted it did it well, and several got the final original part out.

C3.7: Elliptic Curves

Q1 was well answered.

In Q2, few candidates spotted the points of order 4 in Q2(b), and in Q2(d) most candidates got the upper bound on $|y|$ but many had trouble with the upper bound on $|x|$.

In Q3, parts (a),(b) were well answered, but many candidates found part (c) difficult.

C3.8 Analytic Number Theory

Question 1. Attempted by all but one of the candidates, and producing a good range of marks. Many candidates were let down by their manipulation skills in part (b). At the end of part (c) there was a misprint. (“greater than” should have been “greater than or equal to”). However no-one noticed this during the exam, and a careful review of the scripts suggested that none of the candidates had been upset by the error. In general though, few candidates had a clear idea how to attack this final unseen part.

Question 2. This was a successful question in general. However in part (a) some candidates followed the lecture-note route, rather than using the induction approach specified in the question.

Question 3. This was the least popular question. Part (a) which was bookwork, was pretty well done. In part (b) almost all candidates adapted the lecture-note approach, using a zero-free region. While this was successful, it was quite disappointing how few saw that a direct estimation with $\sigma > 1$ led directly to the answer. Part (c) proved too difficult, with no correct answers.

C4.1: Functional Analysis

Question 1 was by far the most popular question and almost all candidates scored most of their marks on it. Questions 2 and 3 appeared harder, but it’s more likely that the difficulties for many candidates were down to lack of time. Question 3 part b was also done on a problem sheet and in light of that it’s remarkable how few candidates got it right.

C4.2: Linear Operators

Q.1: Part (b) caused many more problems than expected. It is basically an algebraic exercise, but it is constrained by the need to ensure that every operator that is written down makes sense as a bounded operator, and most attempts failed to pay attention to that. Half the candidates made good progress with part (c), which was expected to be the most difficult part. No candidate took up the implied offer of carrying out the (very easy) calculation in part (b) for multiplication operators at the cost of being ineligible for full marks, but one candidate tried this in part (c).

Q.2: Part (b) was a modified version of an awkward piece of bookwork from lectures. Although it was somewhat simplified from the lectures, few candidates showed good understanding of the subtleties that arise when dealing with quadratic forms.

Q.3: Only two attempts.

C4.3: Functional Analytic Methods for PDEs

Q1: was the most popular among candidates. Some candidates could not prove density of smooth functions in Sobolev spaces for star-shaped domain. The proof itself is not complicated but based on conceptual properties of mollifications. In the second part of the question, almost all candidates figured out that an approximation by smooth functions should be used but could not justify passing to the limit properly.

Q2: Another popular question was Qn2. There are two things to point out here. The first one is the bookwork question to prove the main estimate for traces in particular case with flat boundary. In addition, not all candidate listed all properties of the extension of the classical trace operator. The other important point is related to Poincaré-Sobolev type inequality in the second part of the question. All candidates mimicked the proof of the Poincaré-Sobolev inequality. The crucial point is to show that the mean value over the boundary of the limit function is equal to zero. This is of course a consequence of continuity of the trace operator in Sobolev spaces.

Q3: Very few candidates attacked the third question although the corresponding attempts were successful. The second part of the question is conceptually more difficult as it is based on density arguments and on two consecutive applications of the uniqueness theorem but technically it is easy.

C4.4 Hyperbolic Equations

Question 1. The first problem is partially based on the material in the past paper or in the notes. Students did well in (a) and (b). (c) requires some observation on nonlinearity. Students can obtain the right identity, but failed to see the correct form of energy for this problem, which is supposed to follow immediately from the identity by direct integration. They still got partial credit by finishing the major part of the energy estimate.

Question 2. (a) is bookwork. (b) was done well. (c) is based on finite speed of propagation and integration by part. There was not much progress done towards this direction.

Question 3. (a)(b) are similar to one problem in the past paper. (c) is related to the proof in notes for the uniqueness of entropy solution. Unfortunately students did not do well in (c).

C4.5 Ergodic Theory

Q1. All candidates attempted this question, which covered the most basic material in the course. It was done quite well. The majority of candidates didn't bother to state which result of Fourier analysis they were using (uniqueness of Fourier coefficients) for which I deducted one mark. In part (b), a common mistake was to equate "positive integer" with "non-negative integer"; this slightly affects the proof so I deducted a mark for this mistake. In part (c), I had not anticipated that some candidates would use a rather trivial (though perfectly correct) example. Of course they got full marks. The majority of candidates did do this part of the question the way I expected. One candidate quoted results about weak-mixing. Unfortunately (though the concept of weak-mixing could certainly motivate one's answer to this question) the question did quite clearly ask for proofs and so I had to

be somewhat harsh on this candidate.

Q2. The first half of this question was simple bookwork and was done quite well. The second part was only done well by a couple of candidates.

Q3. This proved to be a good question - very easy for those who really understood the course, and not so for the others. Several candidates struggled with the second part. Also, a surprising number of candidates omitted to prove that $AP(X)$ is closed under addition, which resulted in the loss of 4 marks.

C4.6 Fixed Point Methods for Nonlinear PDEs

All questions were attempted equally and were generally answered very well by the small but strong group of candidates. As intended the most challenging parts of the questions were 1(b)(ii), 2)(d) and 3(f) for which no complete answers were found, but for which a few good partial attempts were made.

C4.7 Dynamical Systems and Energy Minimization

Question 1: Most of the problem is standard except part (d)(i). Candidates made a mistake in part (c) that “ $u \dot{u} = 0$ in some interval (a, b) implies $u = 0$ in (a, b) or $\dot{u} = 0$ in (a, b) ”. Part (d)(i) was attempted with no success.

Question 2: Parts (a), (b), c(i) and (d)(i) are standard and were handled well. Part (c)(ii) is also standard but candidates had some problem in concluding their answer. Parts (d) (ii) and (iii) are somewhat non-standard but were tried with some success.

Question 3: Part (a) is mostly bookwork; however it showed that candidates had some soft spot about Sobolev spaces and even about Lebesgue spaces. Part (b) is also bookwork but with considerable technicalities which candidates seemed to have reasonable grasp of. Candidates tried part (c) with little success.

C5.1: Solid Mechanics

Q1: Almost all students tried this question and many of them did very well. The question was mostly theoretical and students who learned the material manage to prove most of the statements. Students showed a good understanding on the basics of nonlinear elasticity and were able to manipulate satisfactorily all computations.

Q2: This question was well answered and probably a better test of the students' ability. Most students could do the main basic steps which were similar to homework problems, but only a few students really understood the last steps of the problem and were able to prove the main results.

Q3: This question was probably the easiest. Once set in equations, the computations were straightforward (much shorter than the other two questions) and the students who attempted this problem and understood what was required to do part (c) did very well.

C5.2: Elasticity and Plasticity

Q1: This question was not very popular, with only three used attempts. The boundary conditions at the inner boundary caused some confusion with candidates unsure about whether it is τ_{rr} or $\tau_{\theta\theta}$ that is fixed in part (b); similarly in part (c) only one candidate correctly derived that $\tau_{r\theta}(r = a) = 0$. No candidate addressed the final part of the question.

Q2: This question was very popular. The derivation in part (a) was generally very well done, as was part (b) with many candidates able to justify the ansatz $f(x) \propto \sin n\pi x/L$. In part (c) a number of candidates failed to realise that the free end of the ruler simplifies the problem since then the tension force $T = 0$. Very few were able to solve the eigenproblem for $g(x)$; those who did derive the correct solvability condition for the vibration frequency were able to justify the given inequalities, but did not generally convince that a solution had to exist in that interval.

Q3: This question was very popular, with part (a) almost universally well done. In part (b) a significant number of candidates were unclear about the boundary condition that the displacement field should satisfy, while many others were unable to correctly compute the moment required to impose this displacement field. In part (c), a number of candidates failed to realise that plastic failure occurs at both the top and bottom surfaces, $y = \pm h/2$. This meant that subsequent calculations of the plastic region were often inconsistent.

C5.3: Statistical Mechanics

Q1. 7 attempts, reasonably well done.

Q2. 3 attempts, 1 (very) good.

Q3. 5 attempts, reasonably well done.

C5.5: Perturbation Methods

Question 1. This was a popular question with a broad range of marks. In part (a), a number of candidates lost marks for giving an incomplete inductive argument. In part (b), the application of Laplace's method was reasonably well done, though many marks were lost for failing to justify the size of the error term during each step of the argument or for failing to verify that the expansions are self consistent. The minority of students that chose to quote, prove and apply *Watson's lemma* gave on the whole more deficient arguments than those that chose to proceed directly using the expansion given in the hint. Though a number of candidates identified the correct approach to part (c), only a handful made it to the end.

Question 2. This was another popular question with a broad range of marks. The applications of boundary layer theory in part (a) and of WKB theory in part (b) were well done on the whole. In part (a), marks were lost for failing to apply correctly *Prantl's Matching Principle* or for not explaining why the given expansion was a leading-order additive composite expansion. In part (b), marks were lost for failing to solve correctly the ODEs for ϕ or A_0 , for failing to include or keep track of the constants of integration or for failing to apply correctly the boundary conditions. Only a handful of candidates achieved full marks on part (c), though many received partial credit.

Question 3. This was the least popular question with a less broad range of marks and fewer high marks, though the average was close to that of questions 1 and 2. The applications of regular perturbation theory in part (a) and of multiple scales theory in part (b) were very well done on the whole. In part (a), a number of candidates lost marks for failing to expand $\theta(\varepsilon t)$ as $\varepsilon \rightarrow 0^+$ in the problem for u_1 . In part (b), marks were largely only lost for minor algebraic mistakes. The matching in part (c) and the derivation of the condition for the existence of a constant-amplitude periodic solution in part (d) were well done by only a handful of candidates, though many received partial credit for part (d).

C5.6: Applied Complex Variables

- Q1.** This was a popular question which attracted some good solutions. The sketches of the potential/hodograph planes and the conformal mappings were mostly handled competently. In part (b), many solutions lacked clarity in the application of the Riemann Mapping Theorem, and few candidates correctly identified the average speed with the change in the velocity potential. No-one successfully evaluated the elementary integral required for the end of part (c).
- Q2.** This question was popular, but on the whole not well answered. Most candidates were able to apply the Plemelj formulae correctly in part (a), modulo some confusion over signs. However, part (b) caused a lot of problems. Many candidates failed to justify the solution for the complex potential in the ζ -plane, and were also unable to apply the chain rule correctly. Only two candidates made any headway with part (b)(ii), and no-one correctly determined $\mu(t)$.
- Q3.** This question attracted some very good solutions from those who had mastered this part of the course. The bookwork in part (b) proved unexpectedly challenging, with many unable to quote or apply Cauchy's Integral Formula correctly. Few candidates managed to perform the residue calculations for part (c), but most made at least some progress with the familiar Wiener-Hopf problem in part (d).

C5.7: Topics in Fluid Mechanics

Q1: Parts (a), (b) generally done well - most common errors were errors with by using a plus sign for the pressure instead of $p_x = -h_{xxx}$. Part (c) was hard; a few people got through (i) and very few got the linearised equation right.

Q2: Parts (a), (b) were generally done well. Some people had difficulties introducing the phi factor (that appears in (1)). Part (c) was done by quite a few people, some of which got the similarity equation right. Finding the solution to the ODE was difficult. Part (d) was again easier, in particular quite a few got (5) but some people dropped a mark by giving incomplete answers.

Q3: This questions was somewhat easier; all parts were done by at least one student. Students lost marks where they failed to state a concise criterion for complex characteristics clearly, explain the ill-posedness in terms of a blow-up of the growth rate for large wavenumbers, or calculate D_l correctly.

C5.9: Mechanical Mathematical Biology

Question 1. This question was the least popular but tackled well by all candidates who made a serious attempt.

Question 2. This question had a very wide spread of marks. Candidates generally struggled with determining the Euler Lagrange equations for the example in the question and the Lagrange multiplier often was not eliminated from the final expression for h_0 in part c(i), despite the need to do so.

Question 3. The most popular question, answered by all but one candidate. Generally the question was executed well, though no student recognised that the final part would simplify extensively on noting that oscillatory contributions to the streamfunction do not generate a contribution to the time averaged pumping flux as they average to zero.

C5.11: Mathematical Geoscience

Q1. 4 attempts, not too good but a decent spread of marks. Q3. 5 attempts, a reasonable spread of marks.

C5.12: Mathematical Physiology

This paper was in general well done.

Question 1. This question was reasonably well done.

- A common error in (a)(iii) was to not give all the cases for which one steady state exists.
- In (b)(ii) many candidates failed to deduce that the steady state is linearly unstable, and were then unable to give the criteria for an unstable node and an unstable spiral in terms of ϵ and b .
- Section (b)(iv) was in general very well done. However, very few candidates were able to solve (b)(v).

Question 2. Specific problems encountered by candidates were as follows:

- In (c) some candidates did not use, or justify that, $dU/dX \rightarrow 0$ as $X \rightarrow \pm\infty$.
- (c)(i) candidates did not demonstrate that the remaining steady state was a node or spiral, and many candidates failed to show the dependence of the stability of the steady state on s .
- (d) No candidate was able to do this part.

Question 3. Sections (a) and (b) were generally well done. Candidates found section (c) challenging.

- (c)(i) Some candidates did not use the fact that $E(t_1 - \tau) = \theta$ to deduce t_1 .

- (c)(ii) Some candidates made good inroads into this part of the question, though no candidates gave an explicit expression for t_2 .
- (c)(iii) Several candidates were able to give good answers to this final part.

C6.1: Numerical Linear Algebra

C6.2: Continuous Optimisation

There were some common calculation mistakes and failure to simplify calculations that would have aided some of the solutions. On the other hand, the advantage of the calculation based questions is that they were accessible to all.

C6.3 Approximation of Functions

Q1 Derivation of Chebyshev series coefficients, radius of Bernstein ellipse and application to a function with poles and branch cut. Extremely well done by nearly all who attempted the question, theory well understood, some confusion in last part as to whether the word supremum applied to radius or parameter.

Q2 Characterisation of best approximation, proof of properties of best L-2 approximation in L-infinity norm, application to a particular function. Very well done, both bookwork and problem part.

Q3 Recurrence relation for orthogonal polynomials, application to quadrature. Only a few attempts, three candidates attempted all three questions (generally poorly), those who did attempt the question did not link roots of orthogonal polynomials with quadrature.

C6.4: Finite Element Methods for Partial Differential Equations

Question 1. One candidate scored full marks and several made small slips or omissions. Some did not realise that the term coming from the boundary condition at b was part of the bilinear form (it involves the weak solution u) and could not simply be regarded as an extra term in the linear form (right hand side).

Question 2. The earlier parts were generally well done, but in the significant final part (e), there were some mistakes in vector calculus; several scripts used the divergence of a scalar or similar meaningless quantities.

Question 3. This question was a little more non-standard as it involved a time-dependent fourth order PDE problem. Nevertheless, the majority of serious attempts achieved high scores.

C7.3: Further Quantum Theory

The majority of candidates showed a good understanding of the material and an ability to calculate.

Question 1 was the most popular. Many candidates lost marks by failing to address irreducibility and for being unable to follow through the addition of spin material in detail.

Question 2 was the next most popular. The calculations of the matrix elements were found hard by most and some then ran out of time to give a good answer to the last part.

Question 3 attracted some good solutions, and those that attempted it showed a good understanding although few were able to perform the final integral.

C7.4: Introduction to Quantum Information

Question 1. The average mark achieved was significantly lower than for the other two questions. While all students were well prepared for parts (a) and (b), almost all of them had problems with parts (c) and (d). In particular, the candidates struggled with part (c) where the output state and the probability needed to be computed for the case of an auxiliary qubit in a mixed state.

Question 2. The quality of answers was generally high. Some students got part (a) wrong and did not realize that Alice and Bob will always obtain identical results. Some candidates claimed that obtaining identical results can be used for instantaneous communication. Only few students gave a correct description for establishing a secret cryptographic key in (e).

Question 3. The average mark achieved in this question was the highest among the three questions. Apart from one candidate, marks were above 20 and deductions due to sloppiness or minor mistakes. Surprisingly, not all students could answer part (a) correctly where it was to be shown that the given operation does not allow one to clone a general input state.

C7.5: General Relativity I

Question 1

This question was attempted by the fewest candidates, perhaps because it involved an unfamiliar geometry. Part a) was standard bookwork, although there were some problems explaining why the lagrangian is conserved. Part b) required candidates to combine the conserved quantities in part a) and perform a simple change of variables. The point of part c) was to recognize that the coordinate time t is in fact the proper time of an observer travelling along the time-like geodesic with $\varepsilon = 1$ (a stationary observer at $\rho = 0$). Part d) caused quite a few problems. Students were expected to know that solutions to the equation in part b) are sinusoidal with angular frequency 1 and amplitude $\gamma = \sqrt{1 - \kappa/\varepsilon^2}$. The critical observation for solving the first part of the problem is that the range of the coordinate v is $0 \leq v \leq 1$. In the final part, the proper time may be computed by integrating the expression for the conserved quantity ε .

Question 2

This question guided the students through an unfamiliar derivation of the formula for gravitational redshift in the Schwarzschild solution. Part a) was bookwork and completed well. The hardest part of the problem was the second half of part b): writing down the appropriate 4-vector ζ^a and checking that it obeys the equation from part a). The best answers examined which components required a non-trivial calculation and computed the relevant Christoffel symbol. Some students thought incorrectly that $\nabla_a \zeta^b = 0$ because ζ^b has constant components in the coordinate system of the problem. Part c) was a straightforward application of the normalization $U^a U_a = -1$ for the 4-velocity. The most thorough answers

did not fail to mention that the positive root is chosen to get a future-pointing 4-vector. Full marks for part d) required some discussion of the presence of a horizon.

Question 3

Part a) was designed to test understanding of the physical assumptions underlying the cosmological metric. The best answers demonstrated that τ is the proper time of observers that are ‘stationary’ with respect to the coordinates x, y, z and that these observers measure spatial isotropy and homogeneity. Likewise, the function $a(\tau)$ measures the proper distance between nearby stationary observers. Part b) was bookwork and generally completed well. In part c), almost no-one was precise in explaining the physical meaning of ρ : it is the energy density measured in the rest frame of a stationary observer. The rest of this part was generally completed well. Part d) required the students to compute the age of the universe. The standard line of attack is to convert it into an integral over the scale factor: this was clearly unfamiliar to a number of students. Many students tried to use the approximation $|\Lambda| \ll 3H_0^2$ too early and did not do so consistently. The best answers carefully justified Taylor expanding inside the integral over the scale factor.

C7.6: Relativity II

Overall students did well even though the setter did not think the paper was particularly easy. It seems that the questions were evenly balanced with respect to the marks.

Q1: The setter thought that this would have been the easiest question, however students had a lot of trouble in part (c) (which had been discussed partially in lectures and a question in the problem sheets).

Q2: This was a question on the Kerr black-hole, aiming to find the surface gravity of the Killing horizons. There were good attempts to this question in particular for part (a). There were however many errors in part (b) despite the hint, with only one student completing this part correctly.

Q3: Question 3 was about the Reissner-Nordstrom black hole. It was the more popular and students did well, except for part (d) on the Penrose diagram. Only two students got this last part right, though there were a number on interesting attempts.

C8.1: Stochastic Differential Equations

Question 1. Part (a) consisted mostly of bookwork and was well done by most candidates, most errors appeared in (a)(iv) and are due to the time change. Similarly, Part (b) was well received, the most common error being related again to the time change of Brownian motion in the stochastic integral. The first half of (c)(i) was well done though a couple of students confused linear growth condition of vector fields with boundedness. On the other hand, finding an explicit solution gave a surprisingly large number of students trouble although it was discussed in the lecture and similar examples were part of the exercise sheets.

Question 2. The most common errors in Part (a) were not mentioning that BDG requires local martingales started at 0 and from the localization in (a)(ii). Part (b)(i) was well done by nearly all candidates and most candidates had the right solution attempt for (b)(ii). However, many made calculation errors so that only two students got the correct recursion

formula. No student realized that (c)(i) and the first part of (d) could be done quickly by applying (b)(ii) instead of a longer direct calculation. A common error in (c)(ii) was failing to ensure (by localization) that the conditions for optimal stopping are met.

Question 3. This was very popular and attempted by all 10 candidates; two candidates received the full 25 points. The most common errors in (a) appeared in (a)(iv) due to using a sum $\sum_{i \neq j}$ instead of $\sum_{i < j}$ as a result of not properly checking the induction hypothesis and in (a)(ii) by forgetting the uniform integrability. In part (b) some candidates unnecessarily used the semimartingale decomposition of X, Y which in turn, lead to longer and error prone explicit computations. The most common errors in (c) were just calculus errors (forgetting signs, etc). Few candidates realized that (c)(i) can be done very quickly using that $\langle X^{(\alpha)} \rangle = \alpha^2 \langle X \rangle$.

C8.2: Stochastic Analysis and PDEs

Overall the quality of solutions to all questions was excellent.

Question 1 was the least popular of the three. Nonetheless there were some very good solutions. There was a minor misprint in part (c), where the semigroup should have been $T_t = e^{at(G-I)}$. This did not appear to cause difficulties.

Question 2 was the most popular and there were some extremely good solutions. A number of attempts showed some confusion in the proof of (c), when formally rewriting the conditional expectation given the whole filtration up to time r as an expectation started from the position of the process at time r . In particular, they failed to change the time variable to reflect that the process was ‘restarted’ from that position.

Question 3 was also popular and again there were some excellent solutions. A common mistake was to miscalculate the square of the increment of the proportion of type a individuals in the population process of part (b).

C8.3: Combinatorics

Q1 was generally done very well, although a few candidates got confused between upsets and downsets. Q2 was also done well, and most candidates used the hint to find examples in the last two parts. Q3 was much less popular, although (c) and (d) follow fairly directly from the bookwork in (a) and (b).

C8.4 Probabilistic Combinatorics

Question 1 was attempted by almost all candidates. Part (a) is very standard bookwork, and (b) fairly standard bookwork. These were mostly well done, though often with details wrong or missing. (c)(i) was reasonably well done; (c)(ii) proved difficult, though with some reasonable attempts.

Question 2 was attempted by almost all candidates. The first part is bookwork but quite tricky; there was a range of marks on this, though mostly high. (b) is quite easy and was well done. (c) was generally ok, though with few really complete answers. Surprisingly many candidates reverted to using pairwise independence instead of the relevant dependency

digraph.

Question 3 was only attempted by two candidates. This topic has not come up for a few years in exams, and I wonder whether students gambled that it would not. (When it has come up before, it has been the least popular topic, but not by such a large margin.)

Overall there was a very strong correlation between marks on the different questions.

Statistics Units

Reports on the following courses may be found in the Mathematics and Statistics examiners' report.

SC1: Stochastic Models in Mathematical Genetics

SC2: Probability and Statistics for Network Analysis

SC3: Modern Survival Analysis

SC4: Statistical Data Mining and Machine Learning

SC5: Advanced Simulation Methods

Computer Science

Reports on the following courses may be found in the Mathematics and Computer Science examiners' report.

Quantum Computer Science

Categories, Proofs and Processes

Automata, Logic and Games

Reports on the following courses may be found in the Physics' report.

Theoretical Physics

F. Comments on performance of identifiable individuals

Removed from public version of report.

G. Names of members of the Board of Examiners

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Prof. G-Q Chen
Prof. A Dancer
Prof. C Howls (external)
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